AP Calculus AB Summer Packet

This packet is intended to prepare students for AP Calculus AB by reviewing prerequisite skills from algebra and precalculus. It is due on the first day of school and will count as a quiz grade. At the end of the first week of school, there will be a quiz covering this material.

This packet is lengthy, so please start early. Embedded in this packet are short videos to guide you through the exercises if you get stuck. You can access the videos through Google Classroom. If you need further assistance, please contact me by email bburke@bullochacademy.com and we can schedule an in person meeting or a zoom meeting.

Have a wonderful summer!!

I am looking forward to a great year in calculus!!

-Mrs. Burke

The following formulas and identities will help you complete this packet. You are expected to know ALL of these for the course.

LINES

Slope-intercept: $y = mx + b$

Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$

Horizontal line: $y = b$ (slope = 0)

Vertical line: $x = a$ (slope = undefined)

Parallel → same slope

Perpendicular → opposite reciprocal slopes

QUADRATICS

Standard: $y = ax^2 + bx + c$

Vertex: $y = a(x - h)^2 + k$

Intercept: $y = a(x - p)(x - q)$

Parabola opens: up if $a > 0$
down if $a < 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXPONENTIAL PROPERTIES

$x^a \cdot x^b = x^{a+b}$ $\quad$ $(xy)^a = x^a y^a$

$\frac{x^a}{x^b} = x^{a-b}$ $\quad$ $\sqrt[n]{x^m} = x^{m/n}$

$x^0 = 1 \ (x \neq 0)$ $\quad$ $\left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$

$x^{-n} = \frac{1}{x^n}$

In general, it is fine to have negative exponents in your answers!

LOGARITHMS

$y = \log_a x$ is equivalent to $a^y = x$

$\log_b(mn) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \log_b m$

TRIGONOMETRIC IDENTITIES

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x} \qquad \tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

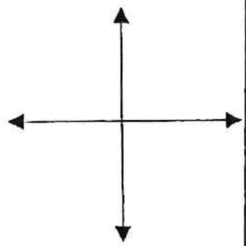
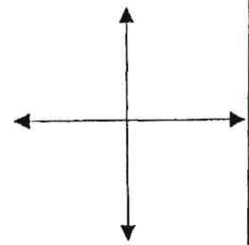
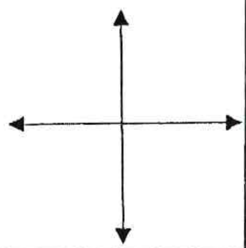
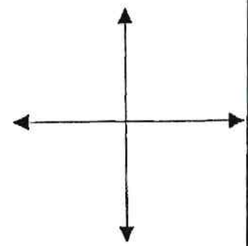
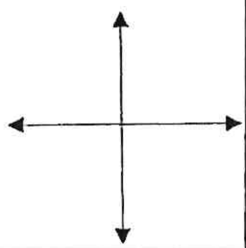
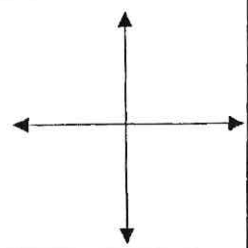
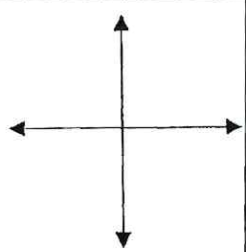
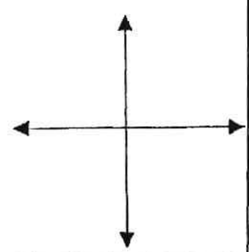
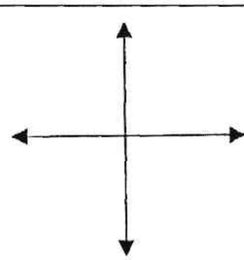
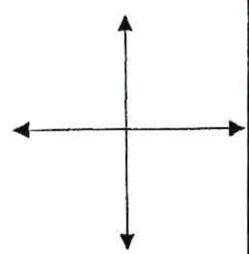
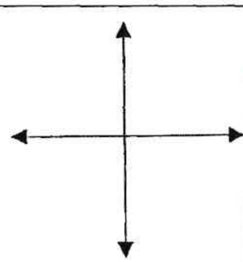
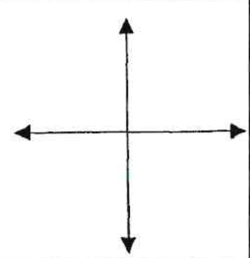
$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2 \sin x \cos x \qquad \cos(2x) = \cos^2 x - \sin^2 x \ \textbf{or} \ 1 - 2\sin^2 x \ \textbf{or} \ 2\cos^2 x - 1$$

Whenever you see a video icon, click it to watch a short video about the content. For example, this video link will help you with the next page.

You are expected to know the general shape, domain, and range of each parent function in the table.

> **"Parent" functions mean no transformations have been applied.**
> Transformation (shifting, stretching, compressing, or reflecting) may change the domain or range.

Linear functions (x) **D:** $(-\infty,\infty)$ **R:** $(-\infty,\infty)$ 	Quadratic functions (x^2) **D:** $(-\infty,\infty)$ **R:** $[0,\infty)$ 
Odd-degree polynomials (x^n for **odd** n) **D:** $(-\infty,\infty)$ **R:** $(-\infty,\infty)$ 	**Even-degree** polynomials (x^n for even n) **D:** $(-\infty,\infty)$ **R:** $[0,\infty)$ 
Exponential functions (a^x, e^x) **D:** $(-\infty,\infty)$ **R:** $(0,\infty)$ Horizontal asymptote: $y=0$ 	Logarithmic functions $(\log_a x, \ln x)$ **D:** $(0,\infty)$ **R:** $(-\infty,\infty)$ Vertical asymptote: $x=0$ 
Sinusoidal functions $(a\sin x, a\cos x)$ **D:** $(-\infty,\infty)$ **R:** $[-a,a]$ 	Tangent function $(\tan x)$ **D:** $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\cup\left(\frac{(2n+1)\pi}{2},\frac{(2n+3)\pi}{2}\right)$ **R:** $(-\infty,\infty)$ Vertical asymptotes: $x=\frac{(2n+1)\pi}{2}$ 
Reciprocal functions $\left(\frac{1}{x}\right)$ **D:** $(-\infty,0)\cup(0,\infty)$ **R:** $(-\infty,0)\cup(0,\infty)$ Vertical asymptote: $x=0$ Horizontal asymptote: $y=0$ 	**Absolute value** functions $(\lvert x\rvert)$ **D:** $(-\infty,\infty)$ **R:** $[0,\infty)$ 
Even root functions $(\sqrt{x}, \sqrt[n]{x}$ for even $n)$ **D:** $[0,\infty)$ **R:** $[0,\infty)$ 	**Odd root** functions $(\sqrt[3]{x}, \sqrt[n]{x}$ for odd $n>1)$ **D:** $(-\infty,\infty)$ **R:** $(-\infty,\infty)$ 

For #1-8, write an equation for each line in point-slope form.

1. Containing $(4, -1)$ with a slope of $\frac{1}{2}$

____

2. Crossing the x-axis at $x = -3$ and the y-axis at $y = 6$

____

3. Containing the points $(-6, -1)$ and $(3,2)$

____

4. Write an equation of a line passing through $(5, -3)$ with an undefined slope.

____

5. Write an equation of a line passing through $(-4,2)$ with a slope of 0.

____

6. Write an equation of a line passing through $(2,8)$ that is parallel to $y = \frac{5}{6}x - 1$.

____

7. Write an equation of a line passing through $(4,7)$ that is perpendicular to the y-axis.

____

8. Write an equation of a line passing through $(6, -7)$ that is perpendicular to $y = -2x - 5$.

____

For #9-16, solve each equation for x. Note that some equations with have a specific value, but most will have a solution in terms of other variables. (For example: $x = \frac{a+b}{c}$ may be a solution.)

9. $x^2 + 3x = 8x - 6$

____

10. $\frac{2x-5}{x+y} = 3 - y$

____

11. $3xy + 6x - xz = 12$

12. $A = ax + bx$

13. $cx = vx$

14. $r = t - x(z - y)$

15. $\frac{3+x}{5-x} = 6 + y$

16. $\frac{y+2}{4-x} = 4(2 - z)$

For #17-22, solve each quadratic by factoring.

This video demonstrates factoring. For the exercises below, you must factor and then solve.

17. $x^2 - 4x - 12 = 0$

18. $x^2 - 6x + 9 = 0$

19. $x^2 - 9x + 14 = 0$

20. $x^2 - 36 = 0$

____

21. $9x^2 - 1 = 0$

____

22. $4x^2 + 4x + 1 = 0$

____

For #23-27, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$.

23. $f\left(\frac{1}{2}\right) =$ __________

24. $g(-2) =$ __________

25. $f(1) + g(0) =$ __________

26. $f(0) \cdot g(0) =$ __________

27. $\frac{g(-6)}{f(-6)} =$ __________

For #28-35, use $f(x) = x^2 - 1$, $g(x) = 3x$, and $h(x) = 5 - x$ to find each composite function.

28. $f(g(x)) =$

____

29. $g(f(x)) =$

____

30. $f(f(4)) =$

____

31. $g(h(-4)) =$

____

32. $f\left(g(h(1))\right) =$

____

33. $f\left(g(x-1)\right) =$ ____________________

34. $g\left(f(x^3)\right) =$ ____________________

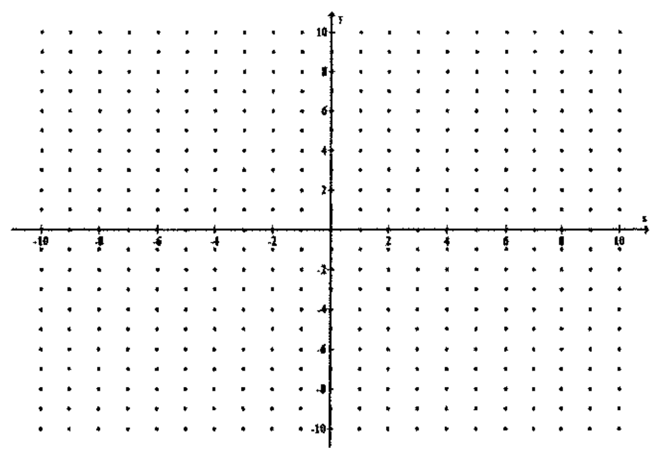
35. $\frac{f(x+h)-f(x)}{h} =$  ____________________

This is an important for calculus.
What is the name of this expression?

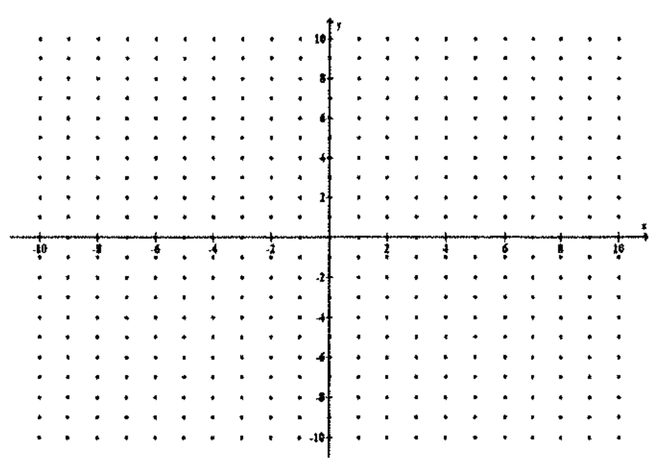
____

For #36-38, graph each piecewise function.

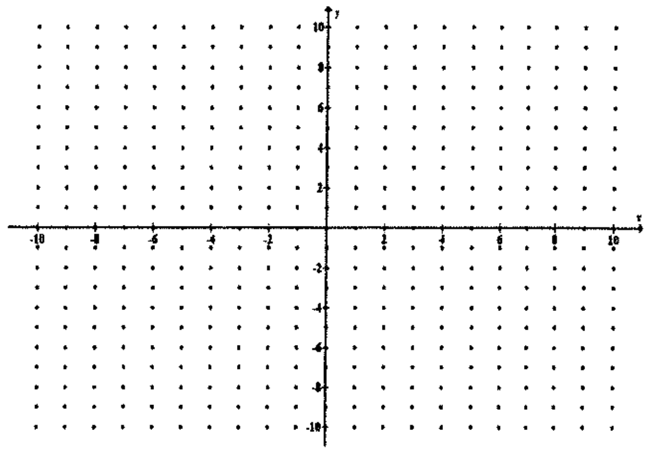
36. $f(x) = \begin{cases} x+3 & ; \ x < 0 \\ -2x+5 & ; \ x \geq 0 \end{cases}$



37. $g(x) = \begin{cases} \frac{1}{2}x & ; \ -4 \leq x \leq 2 \\ 2x-3 & ; \ x > 2 \end{cases}$



38. $h(x) = \begin{cases} |x| & ; \ x \leq 1 \\ 2 - |x-2| & ; \ x > 1 \end{cases}$



For #39-43, solve each exponential equation and round answers to the nearest thousandth. Some equations can be solved by writing each side as the same base while others will require a logarithm.

39. $5^x = \frac{1}{5}$ ____________

40. $6^x = 1296$ ____________

41. $6^{2x-7} = 216$ ____________

42. $5^{3x-1} = 49$ ____________

43. $10^{x+5} = 125$ ____________

For #44-47, simplify each expression without the use of a calculator. The exponential properties on page 2 of this packet will help.

44. $e^{\ln 4} =$ ____________

45. $e^{2\ln 3} =$ ____________

46. $\ln e^9 =$ ____________

47. $5\ln e^3 =$ ____________

For #48-53, solve each exponential or logarithmic equation by hand. Round answers to the nearest thousandth.

48. $e^x = 34$ ________

49. $3e^x = 120$ ________

50. $e^x - 8 = 51$ ________

51. $\ln x = 2.5$ ________

52. $\ln(3x - 2) = 2.8$ ________

53. $2\ln(e^x) = 5$ ________

For #54-66, find the <u>exact</u> value of the expression using the Unit Circle. To be clear, "exact" answer means no decimals!

54. $\sin 120° =$ ________

55. $\cos \frac{11\pi}{6} =$ ________

56. $\tan 225° =$ ________

57. $\sin\left(-\frac{2\pi}{3}\right) =$ ________

58. $\sin 150° =$ ________

59. $\tan \frac{7\pi}{4} =$ ________

60. $\csc\left(\frac{\pi}{4}\right) =$ ________

61. $\sec(-210°) =$ ________

62. $\cot\left(\frac{5\pi}{4}\right) =$ ________

63. $\sin\left(\frac{9\pi}{4}\right) =$ ________

64. $\sec\left(-\frac{\pi}{4}\right) =$ ________

65. $\tan\left(-\frac{4\pi}{3}\right) =$ ________

66. $\cos\left(\frac{8\pi}{3}\right) =$ ________

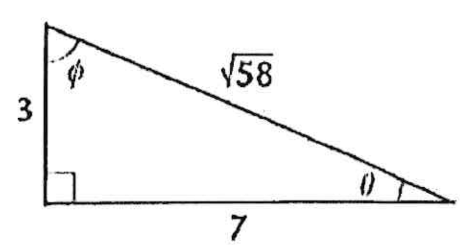
For #67-70, evaluate each trigonometric expression using the right triangle provided. You do <u>NOT</u> need to rationalize the denominator.

67. $\sin\theta =$ ________

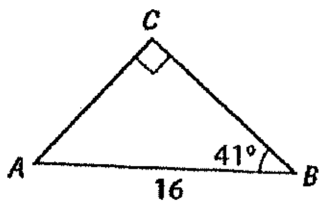
68. $\cos\theta =$ ________

69. $\tan\phi =$ ________

70. $\sec\phi =$ ________



71. Solve the triangle, rounding all angles and sides to the nearest thousandth. ("Solving a triangle" means to find all missing sides and angles.)



$m\angle A =$ ________

$AC =$ ________

$CB =$ ________

For #72-79, evaluate each inverse trigonometric function using the Unit Circle. Write all answer in radians, not degrees. Do not use a calculator.

72. $\sin^{-1}\left(\frac{1}{2}\right) =$ ______

76. $\tan^{-1}(-1) =$ ______

73. $\sin^{-1}(-1) =$ ______

77. $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) =$ ______

74. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ ______

78. $\sin\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right) =$ ______

75. $\tan^{-1}(\sqrt{3}) =$ ______

79. $\sin^{-1}(\cos(0)) =$ ______

80. Explain how the graph of $f(x)$ and its inverse, $f^{-1}(x)$, compare.

____

____

For #81-83, find the inverse of each function.

81. $g(x) = \frac{5}{x-2}$

$g^{-1}(x) =$ ________________

82. $f(x) = \frac{x^2}{3}$

$f^{-1}(x) =$ ________________

83. $y = \sqrt{4-x} + 1$

$y^{-1} =$ ________________

84. If the graph of $f(x)$ has the point $(2,7)$, then what is one point on the graph of $f^{-1}(x)$? ____________

For #85-89, write each inequality in interval notation. For example, $x > 3$ becomes $(3, \infty)$.

85. $1 < x \leq 10$ ____________

86. $x < 0$ or $x \geq 4$ ____________

87. $x \geq -2$ ____________

88. $x \geq 4$ and $x > 10$ ____________

89. $x > 5$ or $x < 7$ ____________

For #90-99, find the domain and range of each function. Write answers in interval notation. Confirm your answer by graphing the function on your calculator. The parent functions on page 3 of this packet will help.

90. $f(x) = \sqrt{x+5}$

D: ____________ R: ____________

91. $g(x) = x^2 - 5$

D: ____________ R: ____________

92. $y(t) = \frac{1}{t+7}$

D: ____________ R: ____________

93. $h(x) = \frac{5}{x^2+1}$

D: ____________ R: ____________

94. $f(x) = \sqrt{x^2+5}$

D: ____________ R: ____________

95. $g(t) = t^3 + 2t - 7$

D: ____________ R: ____________

96. $h(x) = 3\sin(\pi x) - 1$

D: ____________ R: ____________

97. $y(x) = \sqrt[5]{2x+3}$

D: ____________ R: ____________

98. $f(x) = -3e^{2x} + 5$

D: ____________ R: ____________

99. $g(t) = \log_4(x-2) + 1$

D: ____________ R: ____________

For #100-102, find the difference quotient of each function. (Refer back to #35 if needed.)

100. $g(x) = x^2 - 3x$

____

101. $f(x) = \frac{2}{x+1}$

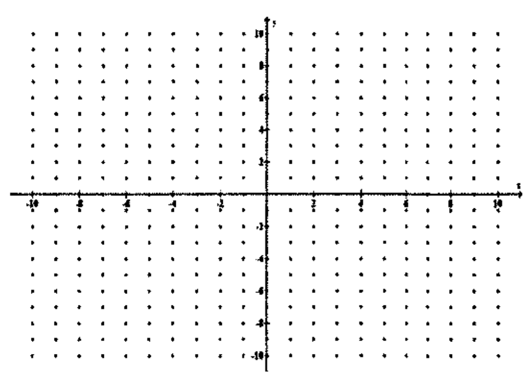
____

102. $h(x) = \sqrt{x-3}$

____

The remaining exercises are more challenging and specifically from concepts and skills covered in Pre-Calculus. You must show all work to earn credit.

103. State the domain and range of $f(x) = \frac{2x^2-6x-20}{x^3-2x^2-15x}$



D: ____________________ R: ____________________

104. Consider the function $f(x) = \frac{e^x}{\log x - x^3}$.

a. Use your calculator to find the relative maximum and minimum y-value of $f(x)$.

min = ____________ max = ____________

b. State the domain of $f(x)$ in interval notation.

D: ____________

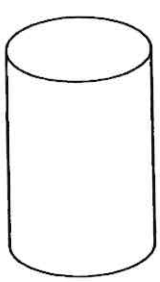
c. State when the function is increasing and decreasing. Write in interval notation.

increasing: ____________ decreasing: ____________

105. A rectangular sheet of tin measures 20 inches by 12 inches. Suppose you cut a square out of each corner and fold up the sides to make an open-topped box. What size square should you cut out in order to maximize the volume of the box? Show all work to earn credit.

____

106. You have been asked to design a cylindrical can that will hold 1000 cubic centimeters. What dimensions (height and radius) will use the least amount of material?



$r =$ ____________ $h =$ ____________