Summer Review for Students Entering Precalculus

Bulloch Academy

This packet is to be handed in to your Precalculus teacher on the first day of school.

All work must be shown in the packet or on a separate sheet of paper attached to the packet.

This packet is worth a major test grade!

Radicals:

To simplify means that 1) no radicand has a perfect square factor and

2) there is no radical in the denominator (rationalize).

Recall the **Product Property** $\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \bullet \sqrt{6}$ find the perfect square factor

$= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}}$ multiply numerator & denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms, multiply the numerator and denominator by conjugate of the denominator (the conjugate of $3+\sqrt{2}$ is $3-\sqrt{2}$)

Simplify each of the following.

1. $\sqrt{32}$
2. $\sqrt{(2x)^8}$
3. $\sqrt[3]{-64}$
4. $\sqrt{49m^2n^8}$
5. $\sqrt{\frac{11}{9}}$
6. $\sqrt{60} \bullet \sqrt{105}$
7. $\left(\sqrt{5}-\sqrt{6}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Rationalize.

8. $\frac{1}{\sqrt{2}}$
9a. $\frac{2}{\sqrt{3}}$
10a. $\frac{3}{2-\sqrt{5}}$

Complex Numbers:

Form of complex number: $a+bi$

Where a is the real part and the bi is the imaginary part

Always make these substitutions $\sqrt{-1}=i$ and $i^2=-1$

To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5}=\sqrt{-1}\bullet\sqrt{5}$ Pull out $\sqrt{-1}$

$=i\sqrt{5}$ Make substitution

Example: $\left(i\sqrt{5}\right)^2=i\sqrt{5}\bullet i\sqrt{5}$

$=i^2\sqrt{25}\ =(-1)(5)=-5$

Treat i like any other variable when $+,-,\times,or\div$ (but always simplify $i^2=-1$)

Example: $2i(3+i)=2(3i)+2i(i)$ Distribute

$=6i+2i^2$ Simplify

$=6i+2(-1)$ Substitute

$=-2+6i$ Simplify and rewrite in complex form

Since $i=\sqrt{-1}$, no answer can have an 'i' in the denominator. RATIONALIZE!

Simplify.

9b. $\sqrt{-49}$

10b. $6\sqrt{-12}$

11. $-6(2-8i)+3(5+7i)$

12. $(3-4i)^2$

13. $(6-4i)(6+4i)$

Rationalize.

14. $\dfrac{1+6i}{5i}$

Geometry:

Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Find the value of x.

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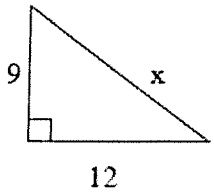

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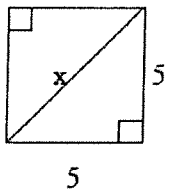

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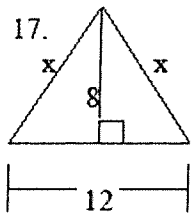
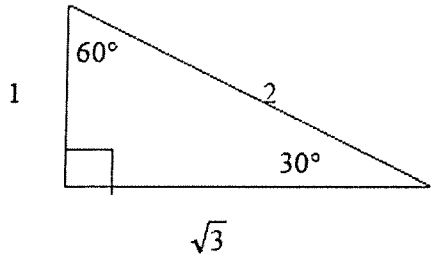
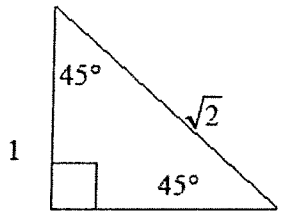


18. A square has perimeter 12 cm. Find the length of the diagonal.

* In $30° - 60° - 90°$ triangles, sides are in proportion $1, \sqrt{3}, 2$.	*In $45° - 45° - 90°$ triangles, sides are in proportion $1, 1, \sqrt{2}$.





Solve for x and y.

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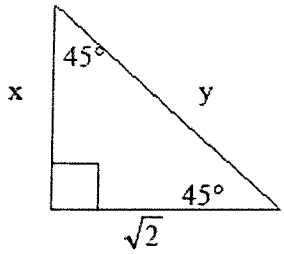

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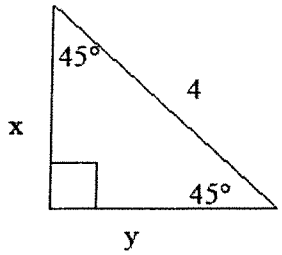

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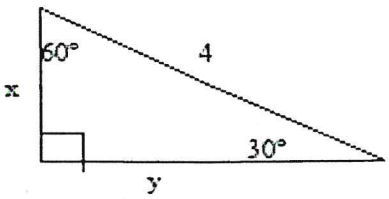

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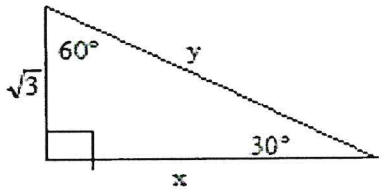


Equations of Lines:		
Slope-intercept form: $y = mx + b$	Vertical line: $x = c$	*(slope is undefined)*
Point-slope form: $y - y_1 = m(x - x_1)$	Horizontal line: $y = c$	*(slope is zero)*
Standard Form: $Ax + By = C$	Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$	

23. State the slope and y-intercept of the linear equation: $5x - 4y = 8$

24. Find the x-intercept and y-intercept of the equation: $2x - y = 5$

25. Write the equation in standard form: $y = 7x - 5$

Write the equation of the line in slope-intercept form with the following conditions:

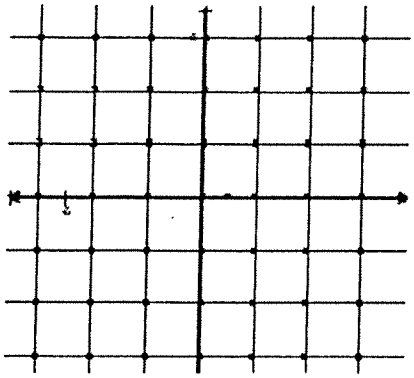
26. slope = -5 and passes through the point (-3, -8)

27. passes through the points (4, 3) and (7, -2)

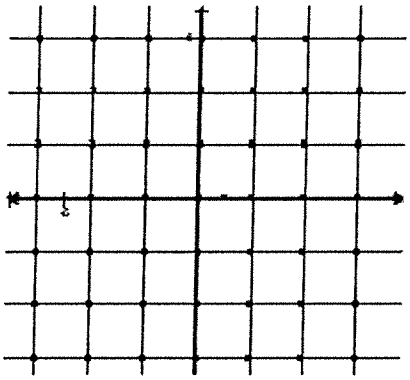
28. x-intercept = 3 and y-intercept =2

Graphing: Graph each function, inequality, and/or system.

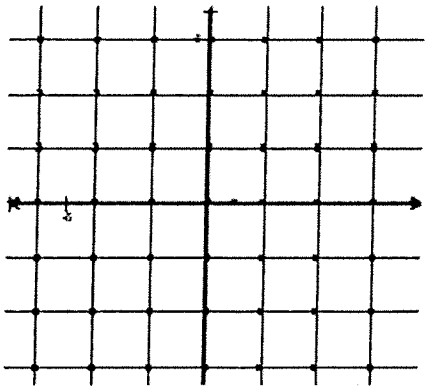
29. $3x-4y=12$



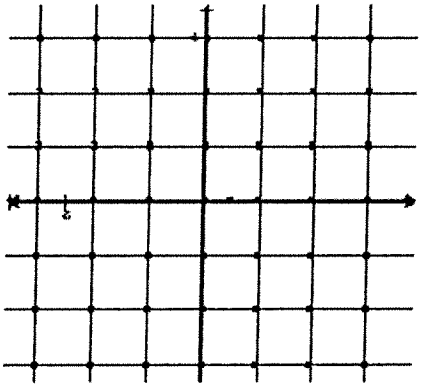
30. $\begin{cases} 2x+y=4 \\ x-y=2 \end{cases}$



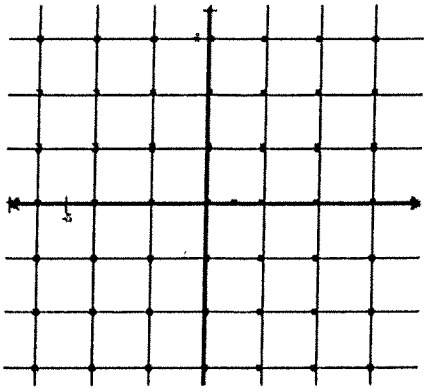
31. $y<-4x-2$



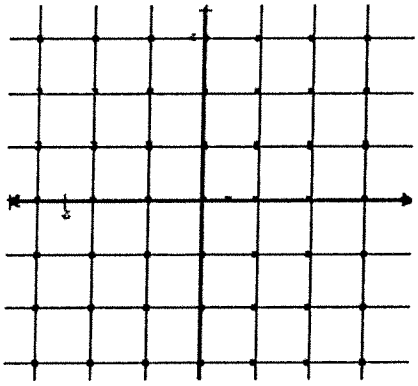
32. $y+2=|x+1|$



33. $y>|x|-1$



34. $y+4=(x-1)^2$



Systems of Equations:

$\begin{cases} 3x+y=6 \\ 2x-2y=4 \end{cases}$

Substitution:	**Elimination:**
Solve 1 equation for 1 variable	Find opposite coefficients for 1 variable
Rearrange.	Multiply equation(s) by constant(s).
Plug into 2nd equation.	Add equations together (lose 1 variable)
Solve for the other variable.	Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$y=6-3x$	Solve 1st equation for y	$6x+2y=12$	Multiply 1st equation by 2
$2x-2(6-3x)=4$	Plug into 2nd equation	$2x-2y=4$	coefficients of y are opposite
$2x-12+6x=4$	Distribute	$8x=16$	Add
$8x=16$ and $x=2$	Simplify	$x=2$	Simplify.

Plug x=2 back into the original equation $6+y=6$, $y=0$

Solve each system of equations, using any method.

35. $\begin{cases} 2x+y=4 \\ 3x+2y=1 \end{cases}$

36. $\begin{cases} 2x+y=4 \\ 3x-y=14 \end{cases}$

37. $\begin{cases} 2w-5z=13 \\ 6w+3z=10 \end{cases}$

Exponents:

Recall the following rules of exponents:

1.	$a^1 = a$	Any number raised to the power of one equals itself.
2.	$1^a = 1$	One raised to any power is one.
3.	$a^0 = 1$	Any nonzero number raised to the power of zero is one.
4.	$a^m \cdot a^n = a^{m+n}$	When multiplying two powers that have the same base, add the exponents.
5.	$\frac{a^m}{a^n} = a^{m-n}$	When dividing two powers with the same base, subtract the exponents.
6.	$(a^m)^n = a^{mn}$	When a power is raised to another power, multiply the exponents.
7.	$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$	Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\frac{3c}{c^{-1}}$

40. $\frac{2ef^{-1}}{e^{-1}}$

41. $\frac{\left(n^3p^{-1}\right)^2}{(np)^{-2}}$

Simplify.

42. $3m^2 \bullet 2m$

43. $\left(a^3\right)^2$

44. $(-b^3c^4)^5$

45. $4m(3a^2m)$

Polynomials:

To add/subtract polynomials, combine like terms.

EX: $8x-3y+6-(6y+4x-9)$ *Distribute the negative through the parantheses.*

$=8x-3y+6-6y-4x+9$ *Combine like terms with similar variables.*

$=8x-4x-3y-6y+6+9$

$=4x-9y+15$

Simplify.

46. $3x^3+9+7x^2-x^3$

47. $7m-6-(2m+5)$

To multiply two binomials, use FOIL.

EX: $(3x-2)(x+4)$ *Multiply the first, outer, inner, and last terms.*

$=3x^2+12x-2x-8$ *Combine like terms together.*

$=3x^2+10x-8$

Multiply.

48. $(3a+1)(a-2)$

49. $(s+3)(s-3)$

50. $(c-5)^2$

51. $(5x+7y)(5x-7y)$

Factoring:

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

a) If you have one (other than 1) factor it out.

b) If you don't have one move on to STEP 2

STEP 2: How many terms does the polynomial have?

2 Terms

a) is it the difference of two squares? $a^2-b^2=(a+b)(a-b)$

EX: $x^2-25=(x+5)(x-5)$

b) Is it the sum or difference of two cubes?

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$
$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

EX:

$$m^3+64=(m+4)(m^2-4m+16)$$
$$p^3-125=(p-5)(p^2+5p+25)$$

3 Terms

	EX:
$x^2+bx+c=(x+\_)(x+\_)$	$x^2+7x+12=(x+3)(x+4)$
$x^2-bx-c=(x-\_)(x-\_)$	$x^2-5x+4=(x-1)(x-4)$
$x^2+bx-c=(x-\_)(x+\_)$	$x^2+6x-16=(x-2)(x+8)$
$x^2-bx-c=(x-\_)(x+\_)$	$x^2-2x-24=(x-6)(x+4)$

4 Terms---Factor by Grouping

a) Pair up first two terms and last two terms.

b) Factor out GCF of each pair of numbers.

c) Factor out front parentheses that the terms have in common.

d) Put leftover terms in parentheses.

$$Ex: x^3+3x^2+9x+27=\left(x^3+3x^2\right)+\left(9x+27\right)$$
$$=x^2(x+3)+9(x+3)$$
$$=(x+3)\left(x^2+9\right)$$

Factor completely.

52. $z^2+4z-12$

53. $6-5x-x^2$

54. $2k^2+2k-60$

55. $-10b^4-15b^2$

56. $9c^2+30c+25$

57. $9n^2-4$

58. $27z^3-8$

59. $2mn-2mt+2sn-2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use the quadratic formula.

EX: $x^2-4x=21$ *Set equal to zero FIRST.*

$x^2-4x-21=0$ *Now factor.*

$(x+3)(x-7)=0$ *Set each factor equal to zero.*

$x+3=0 \quad x-7=0$ *Solve for each x.*

$x=-3 \quad x=7$

Solve each equation.

60. $x^2-4x-12=0$

61. $x^2+25=10x$

62. $x^2-14x+40=0$