Summer Review for Students Entering AP Calculus BC

Bulloch Academy

This packet is to be handed in to your calculus teacher on the first day of school. All work must be shown in the packet or on a separate sheet of paper attached to the packet.

This packet is worth a major test grade

Name: _____ Class: _____ Date: _____ ID: A

AP Calculus BC Summer Packet

Please show all work in the spaces provided.
The answers are provided at the end of the packet.

Algebraic Manipulation

1. Evaluate $\frac{3x-y^2}{3(x-y)}$ if $x=5$ *and* $y=-3$

2. Simplify: $\left(x^{2y}\right)\left(2x^{y}\right)\left(x^{y+3}\right)$

3. $\frac{5x^{-3}y^2}{x^5y^{-1}z^0}\cdot\frac{(2xy^3)^{-2}}{xy}$

4. Simplify $\frac{18xy^3}{7a^2b^2}\div\frac{12x^2y}{35a^2b}$

5. Subtract $(-3g^2+2g-9)$ from (g^2-4g-6)

6. Expand $(3x^4-7y^3)^2$

7. Multiply

   $\left(x^2+x-3\right)\left(3x^2-x+3\right)$

8. Expand using Pascal's Triangle

   $(3x-2)^5$

9. Factor completely

a) $a^2 - 6a - 40$

b) $6y^2 + 13y - 5$

c) $12m^3n - 75mn$

d) $49x^2 - 100y^2$

e) $6xp + 42x - 5yp - 35y$

10. Factor completely

a) $-5x^4y^2 + 20xy^3 + 15xy^4$

b) $3x^3 + x^2 - 15x - 5$

c) $20x^2 - 125y^2$

d) $4x^2 - 12xy + 9y^2$

e) $x^2 - 6x + 9 - 4y^2$

16. Simplify $\dfrac{a^2-x^2}{a^2}\cdot\dfrac{a}{3x-3a}$

17. Simplify.

$$\frac{(4w^2-3wy)(w+y)}{(3y-4w)(5w^2-y^2)}$$

18. Simplify $\dfrac{x^3-64}{x^3+64}\div\dfrac{x^2-16}{x^2-4x+16}$

19. Simplify.

$$\frac{1+\dfrac{1}{x}}{1-\dfrac{x}{y}}$$

20. Simplify the compound fractional expression.

$$\frac{\dfrac{5}{x-1}-\dfrac{4}{x+1}}{\dfrac{x}{x-1}+\dfrac{1}{x+1}}$$

21. Simplify.

$$\frac{5}{6x-18}-\frac{x-1}{4x^2-14x+6}$$

22. Perform the subtraction and simplify.

$$\frac{x}{x^2-x-20}-\frac{1}{x+4}-\frac{3}{x-5}$$

26. Rationalize the denominator and simplify

a) $\frac{2a}{\sqrt[3]{b}}$

b) $\frac{\sqrt[4]{5}}{4\sqrt[4]{27}}$

c) $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{6}}$

d) $\frac{x}{y^{\frac{2}{5}}}$

27. Rationalize the numerator and simplify

a) $\frac{\sqrt{3}+\sqrt{7}}{4}$

b) $\frac{\sqrt{x+3}-5}{3x-66}$

28. Simplify and rewrite using only positive exponents.

$\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

29. Simplify the expression

$-4\left(\frac{2x-1}{2x+1}\right)^{-3}\left[\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right]$

30. $\left(\frac{16}{x^{-2}}-\frac{40}{x^{-1}y^{-1}}+\frac{25}{y^{-2}}\right)^{\frac{-1}{2}}$

Equations and Inequalities

38. Solve $3(x+2)=\frac{1}{4}(12x+4)-5x$

39. Solve the equation.

$$\frac{2}{5}y+\frac{1}{2}(y-4)=\frac{y+1}{4}$$

40. Solve. $\frac{4}{x+3}+\frac{5}{6}=\frac{23}{18}$

41. Solve the equation.

$|3x-5|=7$

42. Solve the equation for the indicated variable:

a) $\frac{ax+b}{cx+d}=11$, solve for x.

b) $\frac{x}{a}=c-\frac{x}{b}$, solve for x.

c) $\frac{x}{a}=c-\frac{x}{b}$, solve for b.

47. Find all solutions of the equation and express them in the form $a+bi$.

$25x^2+16=0$

48. Find all real solutions of the equation.

$x^6-9x^3-10=0$

49. Solve

$4x^{\frac{4}{3}}-37x^{\frac{2}{3}}+9=0$

50. Solve $2(6x-3)^{\frac{1}{3}}-4=0$

51. Solve $\frac{1}{x^2}-\frac{1}{x}=6$

52. Solve

a) $\sqrt{x+2}+4=7$

b) $2=\sqrt{3b-2}-\sqrt{10-b}$

61. Solve the inequality. Graph the solution on a number line.

$|3x+9| \geq 6$

62. Solve and graph your answer on a real number line: $|x-3|-2<6$

63. Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

$x^2+3x>10$

64. Solve the nonlinear inequality. Express the solution using interval notation and graph the solution set.

$\frac{x}{3}-\frac{1}{x-2} \leq \frac{x+1}{4}$

68. Find an equation, in point-slope form, of the line that satisfies the given conditions.

Through (–1, –11); perpendicular to the line passing through (3, 1) and (7, –1).

Then, rewrite the equation in standard form.

69. Solve the system of equations by substitution

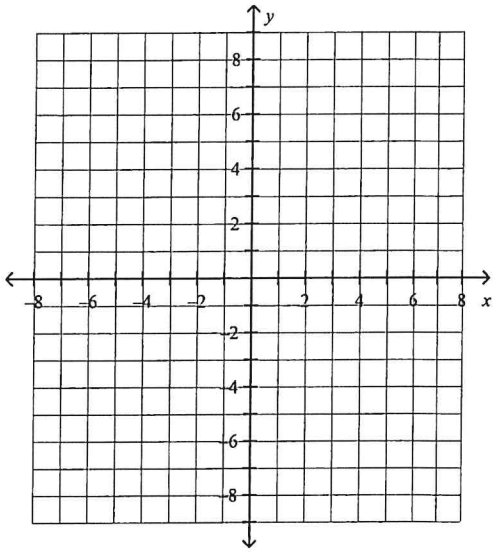
$8x - 2y = 10$

$3x - y = 9$

70. Solve the system of equations by graphing.

$2x + 3y = -27$

$11x - 3y = -12$

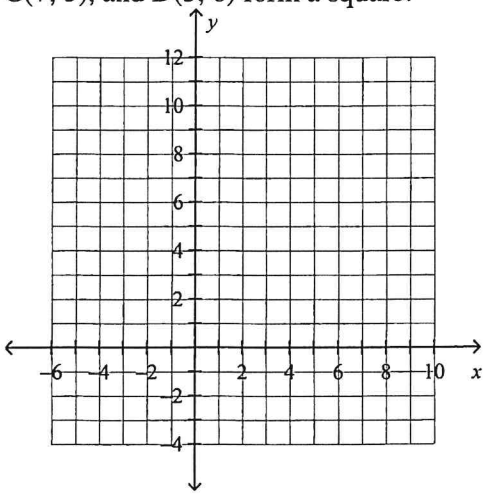


71. Solve the system of equations by elimination

$2x - 3y = 6$

$9y - 6x = 9$

76. Prove whether or not the points A(4, 3), B(2, 6), C(7, 5), and D(5, 8) form a square.



77. Find all points of intersection of the graphs of $x^2 + 3x - y = 3$ and $x + y = 2$.

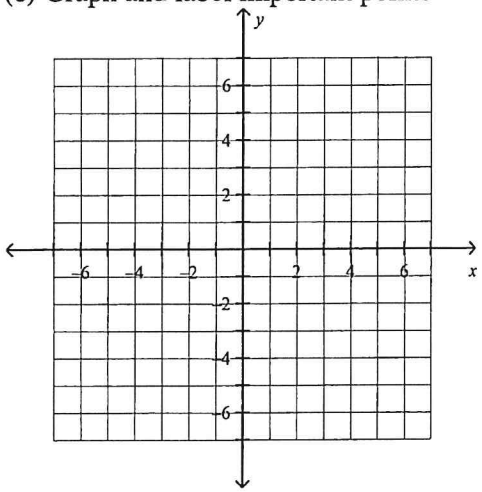
78. Find the center and radius of the circle.

$x^2 + y^2 - 6x + 4y - 3 = 0$

(a) The center is

(b) The radius is

(c) Graph and label important points



(d) Domain:

Range:

79. If the point $(-1,1)$ lies on the graph of the equation $kx^2 - xy + y^2 = 5$, find the value of k.

82. Consider the following function:
$f(x) = 3x^2 - 12x + 7$

a) Write the quadratic function in vertex form.

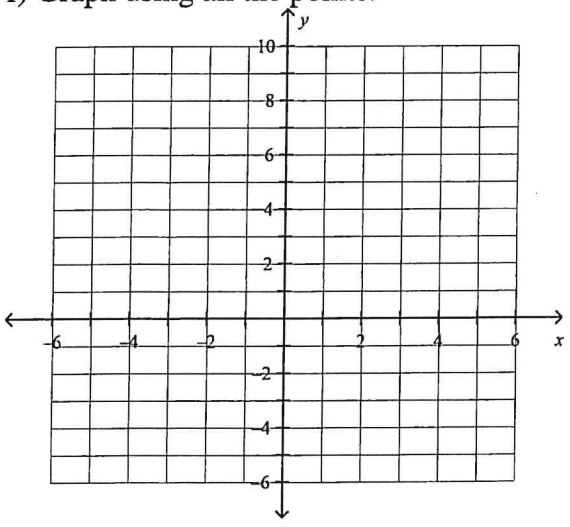
b) State the vertex and whether the graph has a minimum or maximum.

c) State the equation of the axis of symmetry.

d) Find the x intercepts and approximate the values.

e) Find the y intercept

f) Graph using all the points.



83. Consider the following function:
$f(x) = -2x^2 - 12x - 14$

a) Write the quadratic function in vertex form.

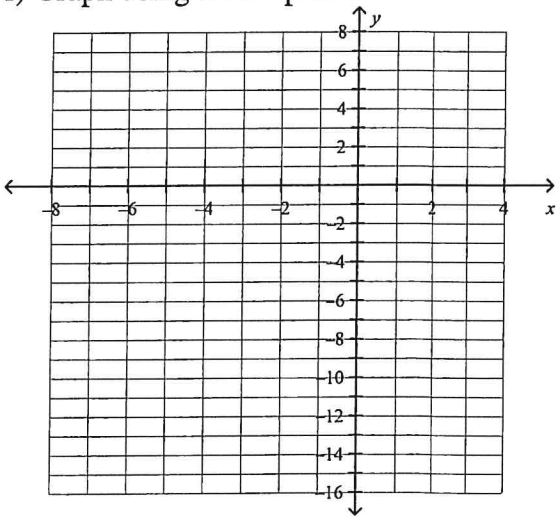
b) State the vertex and whether the graph has a minimum or maximum.

c) State the equation of the axis of symmetry.

d) Find the x intercepts and approximate the values.

e) Find the y intercept

f) Graph using all the points.



91. Evaluate the piecewise defined function at the indicated values.

$$f(x)=\begin{cases} x^2+4x & \text{if } x\le -3 \\ x & \text{if } -3<x\le 1 \\ -9 & \text{if } x>1 \end{cases}$$

(a) Evaluate $f(-4)$.

(b) Evaluate $f\left(-\frac{7}{2}\right)$.

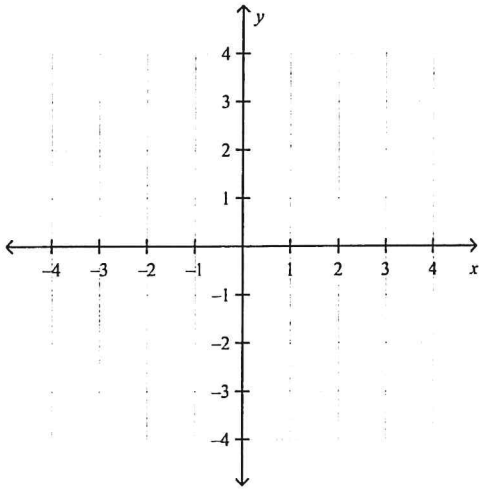
(c) Evaluate $f(-3)$.

(d) Evaluate $f(0)$.

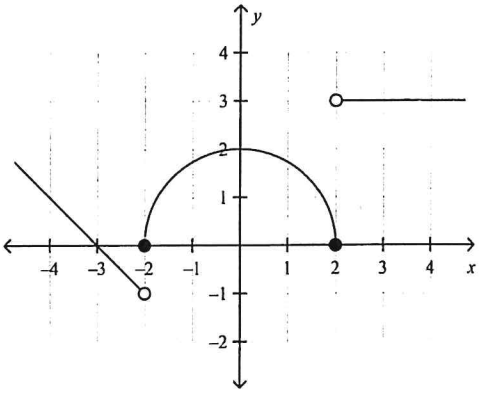
(e) Evaluate $f(35)$.

92. Sketch the graph of the piecewise defined function.

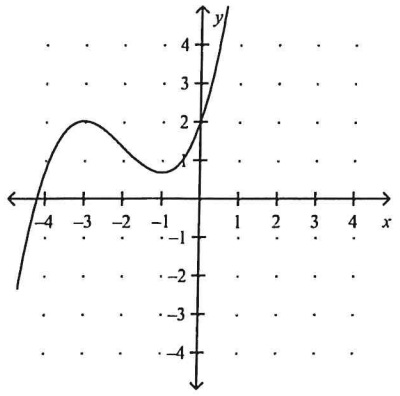
$$f(x)=\begin{cases} 1 & \text{if } x<-1 \\ x^2 & \text{if } -1\le x\le 1 \\ -x+3 & \text{if } x>1 \end{cases}$$



93. Write the equation of the piecewise defined function shown below:



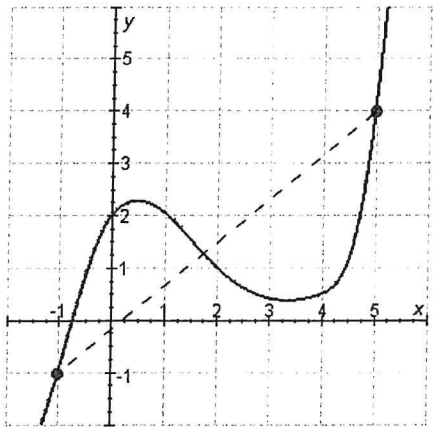
96. The graph of a function is sketched below.



a) Determine the intervals on which the function is decreasing and which it is increasing.

b) State the relative minimum and maximum values.

97. The graph of a function is given. Determine the average rate of change of the function between the indicated values of the variable.



98. Determine the average rate of change of the function between the indicated values of the variable.

$$f(x) = 10 - 6x + x^2$$

a) $x = -1, \quad x = 3$

a) $x = a, \quad x = a + h$

99. Explain what the difference quotient represents in terms of the graph of the function.

124. Use the addition and subtraction formulas to simplify the expression.

$$\frac{\cos(x+y)}{\cos x \cos y}$$

.

125. Use an appropriate half-angle formula to find the exact value of the expression.

$\sin 15°$

.

126. Use the double angle identity to rewrite the expression

$\sin 6x$

.

127. Evaluate in radians

a) $\arccos\left(\frac{-\sqrt{3}}{2}\right)$

b) $\arctan(-1)$

c) $\sin^{-1}\left(-\frac{1}{2}\right)$

d) $\sec^{-1}\left(-\frac{1}{2}\right)$

e) $\arcsin(-1)$

.

128. Evaluate the expression by sketching a triangle.

$\sec\left(\sin^{-1}\frac{12}{13}\right)$

.

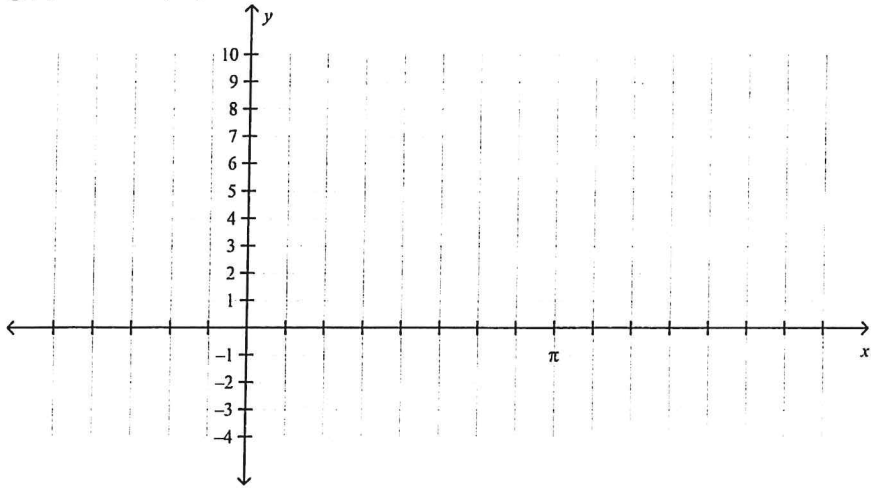
129. Rewrite as an algebraic expression by sketching a triangle.

$\cot\left(\sin^{-1}\frac{x}{7}\right)$

.

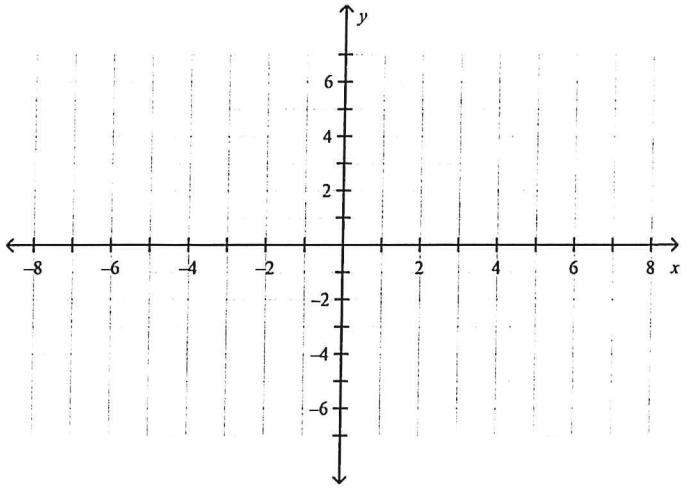
135. Graph the function and state the amplitude, period, domain, and range. Label the values on the x-axis clearly.

$g(x) = -3\sin(2x) + 4$



136. Graph one period of the function and state the period, domain, and range.

$g(x) = 3\tan\left(\frac{\pi}{4}x - \frac{\pi}{2}\right)$



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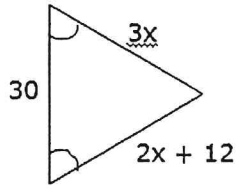
141. A company manufactures and sells small weather radios. If the cost of producting the radio can be expressed as $C(x) = 10,000 + 30x$ and the revenue produced from the sales can be expressed as $R(x) = 50x$, how many radios must be produced and sold for the company to make a profit? (If you don't know the meaning of a term, look it up!)

142. Five times the supplement of an angle is 630° more than its complement. Find the angle, its supplement, and its complement.

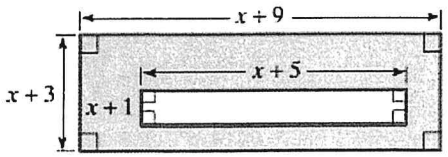
143. Find the value of x. Then, find the perimeter of the triangle.



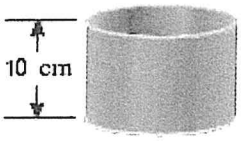
144. Find the area and perimeter of a right triangle with a leg of length 8 and hypotenuse 12.

145. The area of a triangle is 72 in^2 and the base is 8 in. Find the height.

146. In the figure below, find a polynomial expression that represents the area of the shaded region.



150. A cylindrical can has a volume of $90\pi\,\text{cm}^3$ and is 10 cm tall.



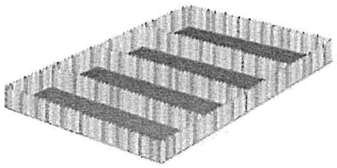
a) What is its diameter?

b) If the can has a bottom, but no top, what is its sufrace area?

151. A poster is 12 inches longer than it is wide. Find a function that models its area A in terms of its width w.

152. The volume of a cone is $80\ \text{in}^3$. Find a function that models the height h of the cone in terms of its radius r.

153. A rancher with 650 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle (see the figure).



(a) Find a function that models the total area of the four pens.

(b) Find the largest possible total area of the four pens.

164. $f(x) = \begin{cases} -x^2 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ x - 2 & \text{if } x > 1 \end{cases}$

Find

a) $\lim_{x \to 1^-} f(x)$ b) $\lim_{x \to 1^+} f(x)$ c) $\lim_{x \to 1} f(x)$

d) $f(1)$ e) Is $f(x)$ continuous at $x = 1$? Why?

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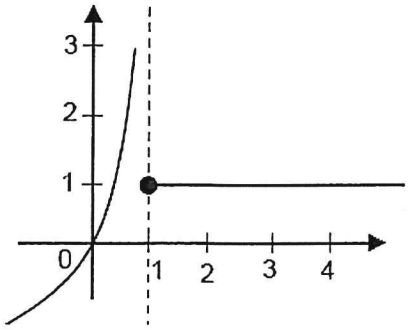
165. The graph of $f(x)$ is shown below.



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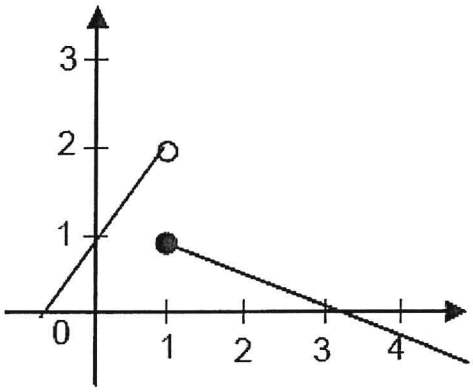

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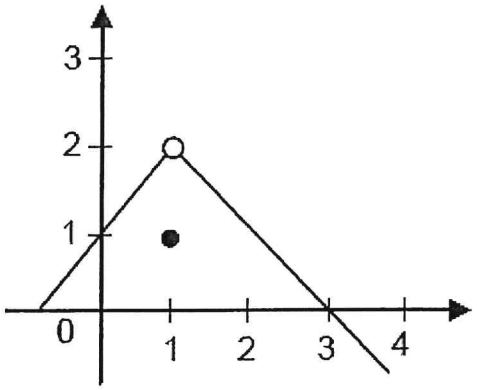

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175. Find the derivative of $f(x) = e^{5x}\cos 7x$.

176. Find the derivative of $g(x) = x^{\frac{3}{2}}\ln x$.

177. Find the derivative of $h(x) = \sin x \cos x$

178. Evaluate $\int_1^{e^2} \frac{1}{x}\,dx$

179. Evaluate $\int_0^9 \frac{1}{\sqrt{x}}\,dx$

180. Evaluate $\int_0^{\pi} (2+\sin\theta)\,d\theta$

181. Evaluate $\int \frac{1+x}{x}\,dx$

182. Evaluate $\int e^{3y}\,dy$

183. Evaluate $\int \cos x\sqrt{\sin x}\,dx$

184. Evaluate $\int \frac{t^2}{t^3+1}\,dt$

195. $f(x) = \int_2^{4x^2} \frac{1}{(t+1)^3}\,dt$

a) Find $f'(x)$

b) Find the equation of the tangent line to $f(x)$ at $x = 1$.

196. Find $\frac{dy}{dx}$ if $y = x^5 \cdot 5^x$

197. Find $\frac{dy}{dx}$ if $y = \ln\sqrt{2x^3 - 6}$

198. Find the slope of the tangent line to the graph of $y = (6\ln x)e^x$ at $x = 4$.

199. Find the particular solution of the equation $f'(x) = 8x^{\frac{1}{3}}$ given that $f(1) = 10$.

200. Find $f(x)$ given that $f''(x) = 4x + 1$ and $f'(1) = 4$ and $f(0) = 3$.

201. An object travels with a constant acceleration of $10\,ft/s^2$. If $v(1) = 8\,ft/s$ and $s(2) = 20ft$, find the position function $s(t)$ describing the motion of this object.

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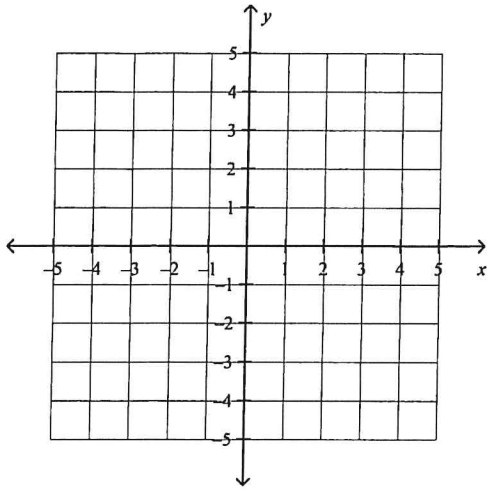
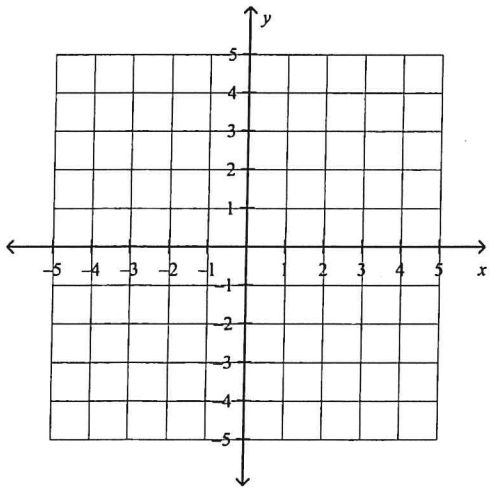
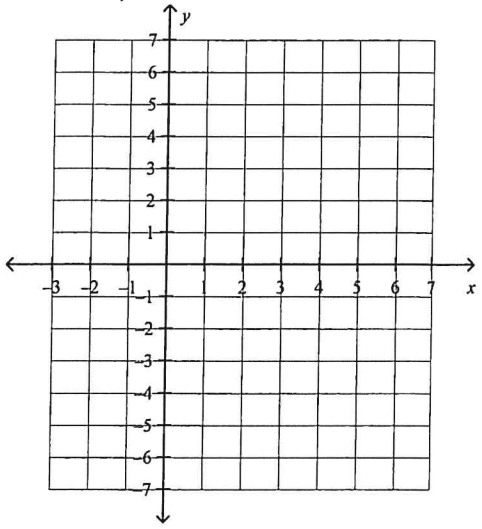
205. Find the area of the region bounded by $y = x^2 - 4x + 5$, the y-axis, $x = 3$ and the x-axis.



206. Find the area of the region bounded by the graphs of $y = 3x - x^2$ and $y = -x$.



207. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = 3x$, $y = 6$ and $x = 4$, around the x-axis.



208. Write, and evaluate using a calculator, a definite integral that represents the volume of the solid formed by revolving the region in quadrant 2 bounded by $y = 4$, $y = \sqrt{x+3}$, $x = -3$, about the line $y = 6$.