Summer Review for Students Entering AP Calculus AB

Bulloch Academy

This packet is to be handed in to your calculus teacher on the first day of school. All work must be shown in the packet or on a separate sheet of paper attached to the packet.

AP Calculus AB Summer Review Packet

Welcome to AP Calculus AB. This packet is a set of problems that you should be able to do before entering this course. You are not required to do EVERY problem in this packet, but you are responsible for all content. I will not collect these problems, but you will be tested on these questions during the first week of school. I'll give you a few days to ask me questions and then you'll have a test on this background material. Make sure you circle any questions that you are unsure about so that you can get clarification during the Q & A period before the test is given.

Probably the biggest difficulty for students is the limitations placed on the use of a calculator. Approximately 70% of the AP exam is without a calculator, so that is the way the course is taught. Approximately 70% of your evaluations will be without a calculator. (That includes a four function calculator, no calculator is allowed at all). When tested on this material, it will be without a calculator.

Finally, I cannot stress enough the importance of these background skills. Calculus is easy, it's the algebra that is hard. Most of the time, you'll understand the calculus concept being taught, but will struggle to get the correct answer because of your background skills. Please be diligent and figure ou what you need help with so that I can clarify for you. A little extra work this summer will go a long wav to help you succeed in the upcoming year.

I look forward to having you in class and working together to accomplish your goals. I think you'll find this course challenging but enjoyable at the same time. Should you have any questions throughout th summer, please feel free to contact me at bburke@bullochacademy.com

I look forward to working with you and having you in class.

Hope you have a great summer!!

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Function

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "f of g of x" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x)=2x^2+1$ *and* $g(x)=x-4$ find *f(g(x)).*

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2+1 \\ &= 2(x^2-8x+16)+1 \\ &= 2x^2-16x+32+1 \\ f(g(x)) &= 2x^2-16x+33 \end{aligned}$$

Let $f(x)=2x+1$ *and* $g(x)=2x^2-1$**. Find each.**

6. $f(2) =$ __________

7. $g(-3) =$ __________

8. $f(t+1) =$ __________

9. $f[g(-2)] =$ __________

10. $g[f(m+2)] =$ __________

11. $\dfrac{f(x+h)-f(x)}{h} =$ __________

Let $f(x)=\sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$ __________

13. $f\left(\dfrac{2\pi}{3}\right) =$ __________

Let $f(x)=x^2$, $g(x)=2x+5$, *and* $h(x)=x^2-1$**. Find each.**

14. $h[f(-2)] =$ __________

15. $f[g(x-1)] =$ __________

16. $g[h(x^3)] =$ __________

Find $\frac{f(x+h)-f(x)}{h}$ for the given function *f*.

17. $f(x)=9x+3$

18. $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

$x-\text{int}.\ (\textit{Let } y=0)$	$y-\text{int}.\ (\textit{Let } x=0)$
$0=x^2-2x-3$	$y=0^2-2(0)-3$
$0=(x-3)(x+1)$	$y=-3$
$x=-1 \text{ or } x=3$	$y-\text{intercept } (0,-3)$
$x-\text{intercepts } (-1,0) \text{ and } (3,0)$	

Find the x and y intercepts for each.

19. $y=2x-5$

20. $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$

22. $y^2=x^3-4x$

Systems

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *and* $x = 5$

Plug $x=3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\quad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\quad$ $16 = y^2$

$y = 0$ $\quad$ $y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *or* $x - 5$

(The rest is the same as previous example)

Find the point(s) of intersection of the graphs for the given equations.

23. $x + y = 8$
$4x - y = 7$

24. $x^2 + y = 6$
$x + y = 4$

25. $x^2 - 4y^2 - 20x - 64y - 172 = 0$
$16x^2 + 4y^2 - 320x + 64y + 1600 = 0$

Interval Notation

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1,7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x-1\geq 0$

28. $-4\leq 2x-3<4$

29. $\frac{x}{2}-\frac{x}{3}>5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x)=x^2-5$

31. $f(x)=-\sqrt{x+3}$

32. $f(x)=3\sin x$

33. $f(x)=\frac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

34. $f(x)=2x+1$

35. $f(x)=\frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \frac{x-9}{4}$ *and* $g(x) = 4x+9$ **show** ***f(x) and g(x)*** **are inverses of each other.**

$$g(f(x)) = 4\left(\frac{x-9}{4}\right)+9$$
$$= x-9+9$$
$$= x$$

$$f(g(x)) = \frac{(4x+9)-9}{4}$$
$$= \frac{4x+9-9}{4}$$
$$= \frac{4x}{4}$$
$$= x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove ***f and g*** **are inverses of each other.**

36. $f(x) = \frac{x^3}{2}$ $\quad g(x) = \sqrt[3]{2x}$

37. $f(x) = 9 - x^2, x \geq 0$ $\quad g(x) = \sqrt{9-x}$

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Radian and Degree Measure

Use $\frac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.	Use $\frac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

46. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

47. Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

48. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians

Reference Triangles

49. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$

b. 225°

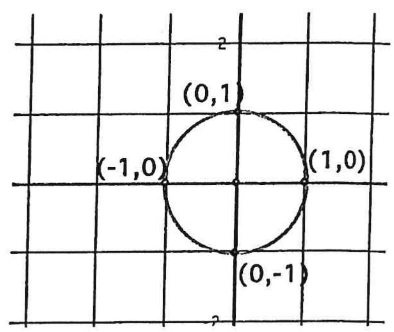
c. $-\frac{\pi}{4}$

d. 30°

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

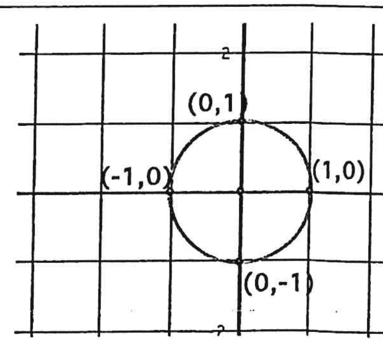
Example: $\sin 90^\circ = 1$ $\cos\frac{\pi}{2} = 0$



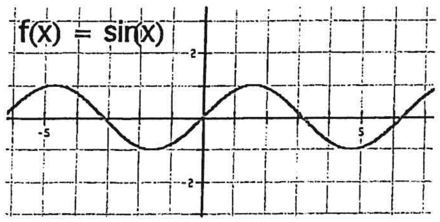
50. $a.)\ \sin 180^\circ$ $b.)\ \cos 270^\circ$

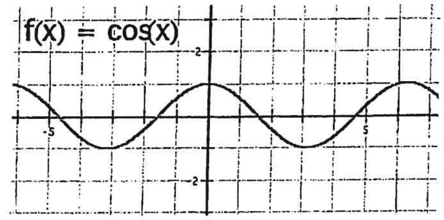
$c.)\ \sin(-90^\circ)$ $d.)\ \sin \pi$

$e.)\ \cos 360^\circ$ $f.)\ \cos(-\pi)$



Graphing Trig Functions





y = sin x and y = cos x have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

51. $f(x) = 5\sin x$

52. $f(x) = \sin 2x$

53. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

54. $f(x) = \cos x - 3$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the beginning of the packet.)

55. $\sin x = -\frac{1}{2}$

56. $2\cos x = \sqrt{3}$

57. $\cos 2x = \frac{1}{\sqrt{2}}$

58. $\sin^2 x = \frac{1}{2}$

59. $\sin 2x = -\frac{\sqrt{3}}{2}$

60. $2\cos^2 x - 1 - \cos x = 0$

61. $4\cos^2 x - 3 = 0$

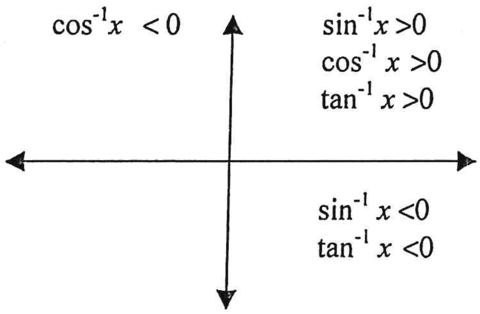
62. $\sin^2 x + \cos 2x - \cos x = 0$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

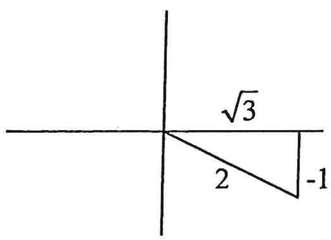
$$\arcsin(x) \qquad\qquad \sin^{-1}(x)$$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:
Express the value of "y" in radians.

$y = \arctan\dfrac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval from

$\frac{-\pi}{2} < y < \frac{\pi}{2}$ Answer: $y = -\frac{\pi}{6}$

For each of the following, express the value for "y" in radians.

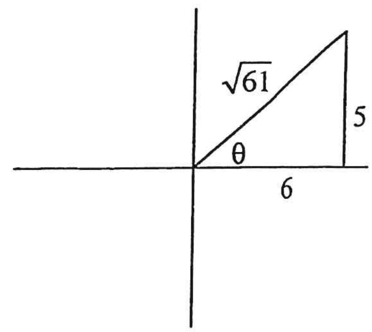
76. $y = \arcsin\dfrac{-\sqrt{3}}{2}$

77. $y = \arccos(-1)$

78. $y = \arctan(-1)$

Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$



Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.

$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

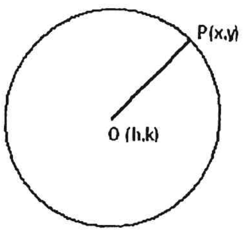
63. $\tan\left(\arccos\frac{2}{3}\right)$

64. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

65. $\sin\left(\arctan\frac{12}{5}\right)$

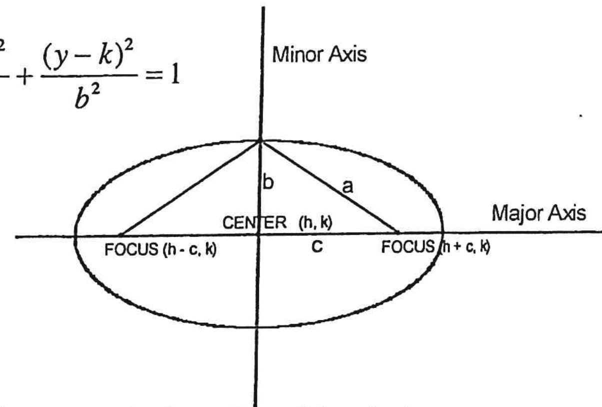
66. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

Circles and Ellipses



$$r^2 = (x-h)^2 + (y-k)^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

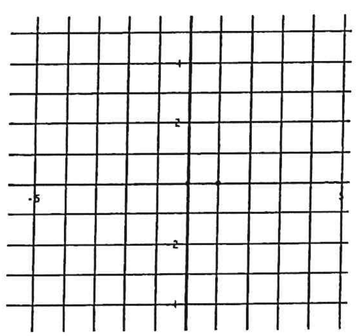


For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where **r** is the radius of the circle.

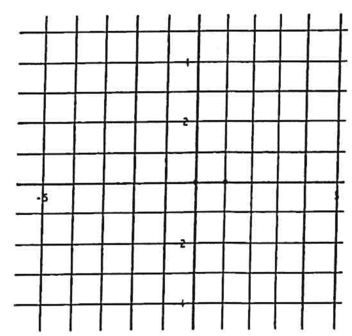
For an ellipse centered at the origin, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where **a** is the distance from the center to the ellipse along the x-axis and **b** is the distance from the center to the ellipse along the y-axis. If the larger number is under the y^2 term, the ellipse is elongated along the y-axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

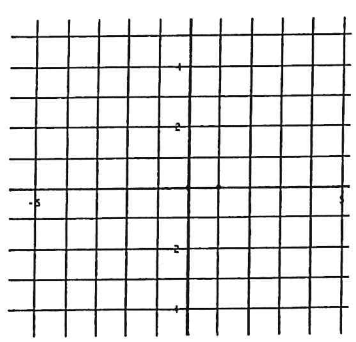
67. $x^2 + y^2 = 16$



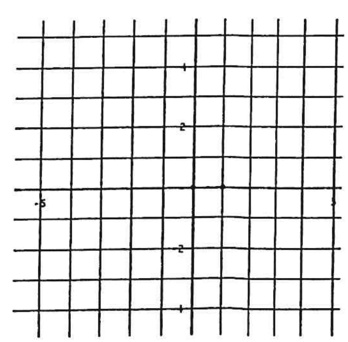
68. $x^2 + y^2 = 5$



69. $\frac{x^2}{1} + \frac{y^2}{9} = 1$



70. $\frac{x^2}{16} + \frac{y^2}{4} = 1$



Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

71. $f(x)=\frac{1}{x^2}$

72. $f(x)=\frac{x^2}{x^2-4}$

73. $f(x)=\frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y=0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

74. $f(x)=\frac{x^2-2x+1}{x^3+x-7}$

75. $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$

76. $f(x)=\frac{4x^5}{x^2-7}$

Laws of Exponents

Write each of the following expressions in the form ca^pb^q where c, p and q are constants (numbers).

75. $\dfrac{(2a^2)^3}{b}$

76. $\sqrt{9ab^3}$

(Hint: $\sqrt{x} = x^{1/2}$)

77. $\dfrac{a(2/b)}{3/a}$

78. $\dfrac{ab-a}{b^2-b}$

79. $\dfrac{a^{-1}}{(b^{-1})\sqrt{a}}$

80. $\left(\dfrac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^2\left(\dfrac{b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)$

Laws of Logarithms

Simplify each of the following:

81. $\log_2 5 + \log_2(x^2-1) - \log_2(x-1)$

82. $2\log_2 9 - \log_2 3$

83. $3^{2\log_3 5}$

84. $\log_{10}(10^{1/2})$

85. $\log_{10}(\frac{1}{10^x})$

86. $2\log_{10}\sqrt{x} + \log_{10} x^{1/3}$

Solving Exponential and Logarithmic Equations

Solve for x. (DO NOT USE A CALCULATOR)

87. $5^{(x+1)} = 25$

88. $\dfrac{1}{3} = 3^{2x+2}$

89. $\log_2 x^2 = 3$

90. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

Factor Completely

91. $x^6 - 16x^4$

92. $4x^3 - 8x^2 - 25x + 50$

93. $8x^3 + 27$

94. $x^4 - 1$

Solve the following equations for the indicated variables:

95. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1,\ for\ a.$

96. $V = 2(ab + bc + ca),\ for\ a.$

97. $A = 2\pi r^2 + 2\pi rh,\ for\ positive\ r.$

Hint: use quadratic formula

98. $A = P + xrP,\ for\ P$

99. $2x - 2yd = y + xd,\ for\ d$

100. $\frac{2x}{4\pi} + \frac{1-x}{2} = 0,\ for\ x$

Solve the equations for x:

101. $4x^2 + 12x + 3 = 0$

102. $2x + 1 = \frac{5}{x+2}$

103. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

Polynomial Division

104. $(x^5 - 4x^4 + x^3 - 7x + 1) \div (x + 2)$

105. $(x^5 - x^4 + x^3 + 2x^2 - x + 4) \div (x^3 + 1)$